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SPECTRAL SHIFT OF LASER RADIATION IN STRUCTURES WITH QUANTUM-SIZE EFFECTS UNDER RESONANT SELF-FOCUSING CONDITIONS

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Based on a semiclassical approach to the analysis of resonant interaction processes between coherent radiation and matter, a computational evaluation of the characteristics of the stationary generation regime in lasers based on quantum-size structures has been performed. The study has shown that the waveguide properties of the gain element and the phenomenon of radiation self-focusing play an important role in the generation process. These effects are driven by the phase resonant nonlinearity of the element's material response, which in turn affects the signal output power.

Keywords: injection lasers, quantum-dimensional structures, resonant nonlinearity of the refractive index, spectral broadening of the absorption line, waveguide properties of the gain element, self-focusing radiation instability.

Introduction

The nonlinear optical model of an injection laser takes into account the influence of free charge carriers, which primarily determine the level of population inversion. This inversion affects the refractive index of the laser material, which, in turn, influences the radiation characteristics. Free carriers play a key role in the laser generation process, as they can modify the optical properties of the material and influence the efficiency of light generation. The consequences of population inversion also include both spectral broadening of the stimulated emission field [1, 2] and its self-focusing instability [3]. Optical losses in the injection laser cavity significantly depend on the waveguide effect, which reduces the diffraction divergence of radiation. The waveguide effect is based on the phenomenon of total internal reflection from a boundary with an optically less dense medium [4]. At higher carrier concentrations, the refractive index decreases; accordingly, the generation region under gain saturation in the laser diode becomes optically denser. This inevitably affects its waveguide properties. It is known that dynamic self-focusing, associated with the nonlinearity of the refractive index, changes the level of compensation for diffraction divergence of radiation [5], thereby influencing the cavity losses. In the literature, the consequences of self-focusing have been studied mainly within the framework of a non-resonant model, where the transverse field distribution is determined primarily by refraction inhomogeneities that inertia-free "track" variations in radiation power. Under conditions of resonant response, the level of nonlinear refraction depends primarily on the degree of inversion in the laser diode.

In this regard, it is of interest to investigate the influence of resonant refraction changes, associated with changes in the energy state of the active layer, on the gain characteristics and dynamics of lasers based on heterostructures with quantum-size effects.

Basic Equations

This paper presents the results of evaluating the consideration of the waveguide properties of the gain element and self-focusing of radiation, stimulated by the resonant nonlinearity of the refractive index in the active layer of the laser diode, in determining the gain. Elementary active centers are considered as two-level dipole quantum emitters (QE). The analysis of the influence of resonant variations in waveguide properties on laser radiation was carried out within the framework of the model substantiated in [6], where non-resonant Kerr nonlinearity was considered for a semiconductor active element. It was assumed that the expression for the real part of the permittivity included a nonlinear component proportional to the stimulated emission field intensity. In essence, model parameters of refractive nonlinearity were used, typical precisely for the consequences of the dynamic refractive index shift observed in semiconductor elements, later characterized as non-resonant [7]. As shown by the results of modern observations, in real lasers with quantum-dimensional effects, including semiconductor quantum dot lasers, refractive nonlinearity has a specifically resonant character [2, 3] and therefore can differ significantly in magnitude and inertia.

We will describe the spatiotemporal characteristics of radiation under conditions of transverse gain and refraction inhomogeneity by considering the two-dimensional representation of the complex dielectric permittivity used in [6] in the form:

$$\varepsilon(x, y, t) = \varepsilon_{nr} - \chi'(x, y, t) + i\chi''(x, y, t), \tag{1}$$

where ε_{nr} is the real (non-resonant) part of the dielectric permittivity of the active layer of the laser element in the unexcited semiconductor. It was assumed that the x direction coincides with the cavity axis; the inhomogeneity of the laser beam cross-section, limited by the waveguide effect, is oriented in the y direction. Furthermore, unlike [6], we assume that the variable real part of the resonant susceptibility (1) is characterized by the usual dispersion dependence:

$$\chi'(x, y, t) = \frac{\mu^2}{3\hbar\varepsilon_0} T_2 N \frac{\Delta_0}{1 + \Delta_0^2} n(x, y, t), \tag{2}$$

where $\Delta_0 = (\omega - \omega_{12})\, T_2$ is the normalized frequency detuning from the center of the gain spectral line ω_{12} and $n(x, y, t)$ is the probability variable of population inversion.

The imaginary component of the resonant susceptibility describing the gain at frequency ω is correspondingly expressed in the following form:

$$\chi''(x,t) = \frac{\mu^2}{3\hbar\varepsilon_0} T_2 N \frac{1}{1+\Delta_0^2} n(x,y,t). \tag{3}$$

Here, in relations (2) and (3), the following notations are used:

- μ is the magnitude of the transition dipole moment;
- T_2 is the transverse relaxation time of the transition; correspondingly, in media used in laser optics of semiconductors, this parameter characterizes the intraband relaxation time;
- the spectral width of the resonant gain line is determined by the factor $2/T_2$;
- N is the volume density of elementary QE.

In [6] a certain approximation of the transverse (in the y-coordinate direction) inhomogeneity of the real part of the permittivity was introduced. We apply an analogous form of inhomogeneity for the probability variable of inversion: $n(x, y, t) = n_m(x, t)$ $[\exp(\delta y) - 2\exp(\delta y/2)]$. In this expression, δ characterizes the width of the active layer, and $n_m(x, t)$ determines the amplitude values of the real and imaginary components of the resonant susceptibility. The form of the spatial distribution $n(y, t)$ during generation (over time) is assumed to be unchanged. In the longitudinal direction (along the x-coordinate), the change in $n(x, t)$ is due to the intensity growth and the corresponding saturation of population inversion. Then, using the methodology outlined in [6], we express the magnitude Ω of the imaginary addition to the frequency of the longitudinal cavity mode:

$$\Omega = \frac{\omega}{2\varepsilon_{nr}}\left[\chi''_m - \chi''_{m0} - \frac{2\delta c}{\omega}\sqrt{\frac{1}{2}\left(\sqrt{\chi''^2_m + \chi'^2_m} - \chi'_m\right)}\right]. \tag{4}$$

The imaginary addition (4) is averaged over the field localization region and essentially expresses the gain coefficient of the laser diode, determining the rate of intensity growth at frequency ω in the active layer. The initial value χ''_{m0} is proportional to the initial value of inversion n_0. Further, analogously to [8], we take into account the influence of the quasi-resonant component of susceptibility on the real part of permittivity. This component is associated with radiation absorption in transitions adjacent to the resonant one and formally depends on the difference in polarizabilities of elementary QE at the levels of the main transition. It is precisely this contribution that primarily determines the refractive nonlinearity of the active element measured in [5] and the associated automodulation frequency shift of generation in semiconductor lasers. In the scientific literature, it is often called the spectral broadening of the field [9], associated with the dynamics of the so-called Henry factor. The magnitude of spectral broadening is proportional to the resonant variation of inversion and depends linearly on $\Delta\alpha$ – the difference in polarizabilities of QE at the levels of the main transition.

Besides, in the expression for the nonlinear frequency addition, it is advisable to consider the shift of the resonant transition frequency initiated by the polarizing influence of near fields of dipoles. Its contribution to the change in frequency detuning has been measured and can be estimated by taking into account the local Lorentz correction [10] to the effective field acting on the QE. The dynamic factor of frequency shift, which is due to dipole-dipole interaction, is considered particularly significant in the radiation processes of quantum dot lasers [11, 12]. Its representation in subsequent computational estimates is used the same as in [8].

Taking into account both mechanisms of phase nonlinearity, we write the expression for frequency variation (4) as follows:

$$\Omega = \frac{\mu^2 N\omega}{6\varepsilon_0\hbar\varepsilon_{nr}} T_2\left[n_m\big/(1+\Delta^2) - n_0 - \frac{2\delta c}{\omega}\sqrt{\frac{1}{2}\left(\sqrt{n_m^2\big/(1+\Delta^2)^2 + \vartheta^2} - \vartheta\right)}\right] \tag{5}$$

$$\vartheta(n_m) = n_m\Delta\big/(1+\Delta^2) - \beta(n_0 - n_m), \quad \Delta = \Delta_0 + \gamma n_m, \quad \beta = 2\pi\Delta\alpha\varepsilon_0\hbar\big/\mu^2 T_2, \quad \gamma = \mu^2 N T_2\big/3\varepsilon_0\hbar.$$

Here, in the expression for the modified resonant addition $\gamma(n_m)$, there is a coefficient β, which relates the change in refractive index to the variation in the concentration of dipole QE. The parameter γ defines the normalized coefficient in the local Lorentz correction.

Modeling of the stationary generation regime of lasers based on quantum-size structures

Let us analyze the stationary laser model. It is based on nonlinear transfer equations for counterpropagating waves with steady-state field strength amplitudes $E_\pm$, forming one of the cavity modes — a standing light wave, known, for example, from [13]. The inclusion of resonant addition (5) in the calculation system, as in [6], means taking into account additional losses in the cavity. These losses are associated with changes in radiation divergence due to nonlinear refraction. For normalized field intensities $S_\pm(x) = \mu^2\tau_1 T_2\left|E_\pm(x)\right|^2\big/\hbar^2$, where τ_1 is the longitudinal relaxation time or, in the case of laser diodes, spontaneous recombination time, the equations of the computational scheme taking into account (5) are written as:

$$\frac{dS_\pm}{dx} = \kappa\left[\frac{n_{ms}}{1+\Delta^2} - \rho\sqrt{\sqrt{\left(\frac{n_{ms}}{1+\Delta^2}\right)^2 + \vartheta_s^2} - \vartheta_s} - \sigma\right] S_\pm,$$

$$\gamma_s = \left[\Delta - \beta\left(S_+ + S_-\right)\right]\Big/\left(1 + \Delta^2 + S_+ + S_-\right), \tag{6}$$

$$n_{ms} = 1\Big/\left[1 + \left(S_+ + S_-\right)\Big/\left(1+\Delta^2\right)\right], \quad \kappa = \frac{\mu^2 N\omega}{3\varepsilon_0\hbar\eta c}T_2 n_0, \quad 0 \le x \le l,$$

where κ is the gain coefficient achieved due to the pump current, σ is the relative parameter of linear losses, ηl is the optical length of the active layer, $\rho = \delta c\sqrt{2}/\omega$.

Equations (6) were solved with boundary conditions corresponding to reflection conditions at the left facet ($x = 0$) with a fully reflecting mirror, and at the right facet ($x = l$) with partial reflection (energy reflection coefficient $r = 0.32$):

$$S_+(x=0) = S_-(x=0), \quad S_-(x=l) = rS_+(x=l). \qquad (7)$$

For conditions (7), the system of equations (6) is analytically solvable only partially. Namely, for the functions $S(x) = S_+(x) + S_-(x)$ and $R(x) = S_+(x) - S_-(x)$, a simple solution in quadratures is possible: $S^2(x) - R^2(x) = 4r\,R(l)/(1-r)^2$. This relation reduces the problem of determining the dependence $S(x)$ to solving one nonlinear differential equation, which is convenient to write in the form:

$$\kappa\frac{dx}{dS} = \frac{1+\Delta^2+S}{F\left\{1-\rho\sqrt{\left(1+\Delta^2+S\right)\left[\sqrt{1+\left(\Delta-\beta S\right)^2}-\Delta+\beta S\right]}-\sigma\left(1+\Delta^2+S\right)\right\}}, \qquad (8)$$

$$F = \sqrt{S^2 - 4rR^2(l)/(1-r)^2}, \quad (1+r)R(l)/(1-r) \le S \le 2\sqrt{r}R(l)/(1-r).$$

Considering the quantity S as an independent variable (non-negative and linearly increasing), one can perform numerical integration of (8) within the indicated limits. These limits are determined by the values of the power flux at the reflecting and "output" facets. The parametric integration of the original system (6), simplified in this way, makes it possible to obtain a numerical solution of (8) with respect to the gain parameter κl and to construct the dependence of the output power flux $I = (1-r)\,Y_+(x=l)$ on the pump parameter $\alpha = \kappa l/(\sigma l - \ln(r)/2)$.

Figure 1 presents the results of the numerical modeling of the system of equations (8), illustrating the output intensity $I(\alpha)$ as a function of the pump parameter for different values of the transverse inversion inhomogeneity parameter and frequency detuning.

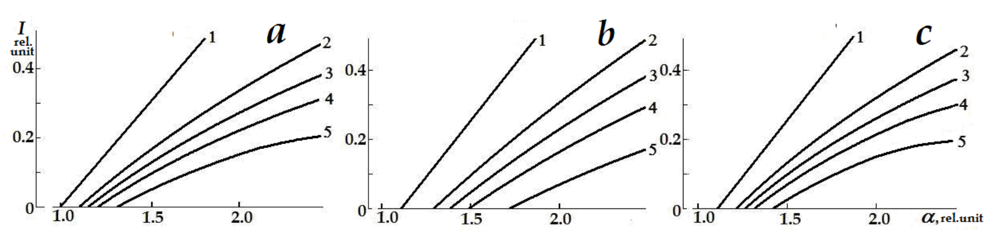

Figure 1. Output intensity as a function of the pump parameter for various values of the transverse inversion inhomogeneity parameter and frequency detuning:

$\rho = 0$ (curve1), 0.1 (2), 0.14 (3), 0.18 (4), 0.25 (5); $\Delta_0 = 0$ (*a*), –0.5 (*в*), 0.5 (*c*); $\beta = 0.15$, $\gamma = 1.58$, $\omega = 1.5 \cdot 10^{15}$ *рад/с*, $\sigma = 0.2$, $\eta l = 1.0 \cdot 10^{-3}$м, $\tau_1 = 1.0 \cdot 10^{-9}$*с*, $T_2 = 1.0 \cdot 10^{-12}$*с*.

In constructing the dependencies shown in Figure 1, the corresponding coefficients of system (6) were determined based on injection laser parameters known from the scientific literature, for example, [2, 14]. As can be seen from the plots in Figure 1, with a more pronounced refractive index inhomogeneity (i.e., larger values of ρ), the effective gain level required for the onset of lasing must increase. It is evident that the output power curves originate from values greater than 1, which corresponds to the threshold value for the exact resonance frequency $\omega = \omega_{12}$. The threshold values naturally increase as the linear frequency detuning Δ_0. This can be observed by comparing the behavior of the curves in panels *a-c* of Figure 1. Thus, an increase in both of these factors – inhomogeneity and frequency detuning – will also require a relative increase in the pump current for lasing to develop. As the inhomogeneity parameter increases, the output power decreases, and the curves in Figure 1 become increasingly nonlinear. Furthermore, we note an asymmetry in the growth behavior of the $I(\alpha)$ curves in Figure 1 depending on the sign of the frequency detuning Δ_0.

This trend is more clearly demonstrated by the curves in Figure 2. These illustrate the dependencies of the normalized output radiation intensity on the mode frequency at a certain above-threshold pump level α. Such curves can be used to characterize the spectral dependence of the gain.

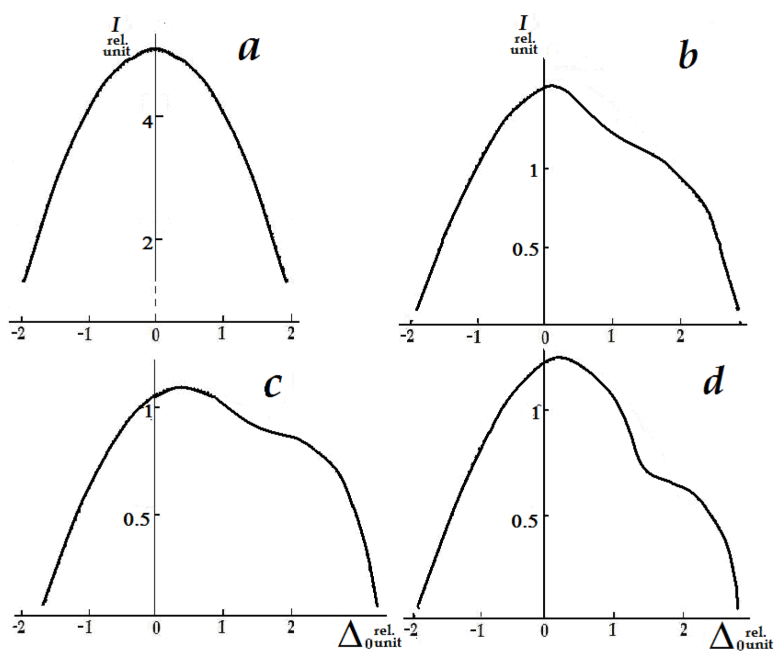

Figure 2 - Spectral dependence of the output intensity for various values of the transverse inversion inhomogeneity parameter and the resonant nonlinearity parameter:

$\rho = 0$ (a), 0.12 (b), 0.2 (c, d); $\beta = 0$ (a) 0.1 (b, c), 0.15 (d) $\omega = 1.5{\cdot}10^{15}$ *рад/с*,

$\sigma = 0.2$, $\gamma = 1.58$

$\eta l = 1.0{\cdot}10^{-3}$м, $\tau_1 = 1.0{\cdot}10^{-9}c$, $T_2 = 1.0{\cdot}10^{-12}c$.

The natural shape of the symmetric gain resonance is shown in Figure 2 (a) and was calculated in the absence of nonlinear refraction. Panels *b-d* of Figure 2 demonstrate quasi-resonant curves with a short-wavelength shift of the maximum and asymmetric broadening. This deformation of the radiation characteristics, presented in panels *b-d* of Figure 2, arises from changes in the light loss level due to self-focusing. Furthermore, panels *b-d* of Figure 2 show that the deformation of the radiation characteristics depends on the parameters of transverse inhomogeneity and resonant nonlinear refraction.

Conclusion

In this work, a theoretical investigation of the dynamic characteristics of the steady-state generation regime of injection lasers based on quantum-confined structures has been conducted within the framework of a semiclassical approach, taking into account the resonant nonlinearity of the refractive index in the active layer. A computational model has been formulated that incorporates two mechanisms of phase nonlinearity: the quasi-resonant susceptibility component arising from the difference in polarizabilities of quantum emitters at the fundamental transition levels, and the resonant frequency shift due to dipole-dipole interaction with consideration of the local Lorentz correction. Based on nonlinear transfer equations for counter-propagating waves, a system of differential equations describing the steady-state laser generation regime has been derived and numerically solved, accounting for the transverse inhomogeneity of the population inversion.

Numerical modeling has revealed that an increase in the transverse inhomogeneity parameter of the refractive index leads to an elevation of the threshold gain level and a reduction in the output radiation power. A pronounced asymmetry in the spectral gain characteristics has been identified: resonant self-focusing gives rise to a short-wavelength shift of the spectral dependence maximum and its asymmetric broadening. The obtained results indicate that the spectral broadening of the generation field in lasers based on quantum-confined structures may, to a certain extent, be attributed to the self-focusing effect associated with the phase nonlinearity of the resonant material response, which is essential for the correct interpretation of experimental data and the optimization of injection laser parameters.

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Тимощенко Е. В. **СПЕКТРАЛЬНОЕ СМЕЩЕНИЕ ИЗЛУЧЕНИЯ ЛАЗЕРОВ НА СТРУКТУРАХ С КВАНТОВОРАЗМЕРНЫМИ ЭФФЕКТАМИ В УСЛОВИЯХ РЕЗОНАНСНОЙ САМОФОКУСИРОВКИ**

На основе полуклассического подхода к анализу процессов резонансного взаимодействия когерентного излучения с веществом проведена расчетная оценка характеристик стационарного режима генерации лазеров на квантоворазмерных структурах. Исследование показало, что волноводные свойства усиливающего элемента и явление самофокусировки излучения играют важную роль в процессе генерации. Эти эффекты стимулируются фазовой резонансной нелинейностью материального отклика элемента, что в свою очередь влияет на выходную мощность сигнала.

Ключевые слова: инжекционные лазеры, квантоворазмерные структуры, резонансная нелинейность показателя преломления, спектральное уширение линии поглощения, волноводные свойства усиливающего элемента, самофокусировочная неустойчивость излучения.