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RELAXATION STRUCTURE OF LIGHT REFLECTION BY A LOW-DIMENSIONAL ARRAY OF RESONANT QUANTUM EMITTERS

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A modification of the Maxwell-Bloch system of equations is proposed for studying oscillations in the intensity of coherent radiation reflected from a low-dimensional array formed by a structure of relatively high-density resonant dipole centers. Modeling and qualitative analysis of the proposed system of kinetic equations showed that variations in the phase relationship of the light field and the resonant polarization of the array cause internal instability of the physical energy exchange system, manifested in a pronounced oscillatory structure of the reflected radiation. The modeling utilized the material properties of semiconductor quantum dots.

Keywords: quantum-dimensional structures, dipole-dipole interaction, resonant reflection, optical bistability, self-pulsation of radiation.

Introduction

Optical structures made of extremely thin layers based on media with a nonlinear response in the frequency domain of optical resonance represent an example of the simplest physical system that can be modeled for studying the dynamics of energy exchange between the electromagnetic field of light and layers of active material. Models of their interaction are used to study optical bistability and self-pulsations of light, the generation and transformation of the structure of coherent radiation, and even the emergence of dynamic chaos in it [1–5]. Practical interest in such objects is stimulated by the possibilities of their application in compact optoelectronic devices, including the development of materials capable of effectively changing the phase characteristics and direction of reflected light fluxes (thin layers with the properties of metasurfaces) [3-6].

The amplification or generation of light upon reflection by an inversion layer or a low-dimensional array of resonant centers (RCs) with a population inversion remains a pressing problem, although researchers have addressed it at various times [7, 8]. It was predicted, in particular, that upon total reflection from an inverted medium, the reflected light flux could be amplified. The possibility of its amplitude modulation due to the nonlinearity accompanying such interaction of the light field with the active medium was not considered; it was rightly believed that the known nonlinearity of the medium's response, caused by the thermal action of radiation, is characterized by high inertia. Artificial electromagnetic composite media with optical properties absent in the original natural materials of which they are composed are currently being intensively developed. These initially non-obvious optical properties are due to the subwavelength size of the elementary dipole emitter during their dense packing in the medium and the resonant nature of their response to an external light field. The nonlinearity of the medium's response, which manifests itself at the resonance frequency, is particularly significant in semiconductor structures with quantum-dimensional effects, which have been intensively studied in recent years [9, 10]. These structures are classified among the above-mentioned composite materials with a high density of active centers. In the exciton region of the spectrum, the probability of induced transitions and a resonant response to a coherent light field is particularly high there [11] – so much so that these materials are usually cited as an example of dense resonant media with their inherent, virtually inertialess property of phase nonlinearity, which is expressed in the frequency drift of the resonant spectral line, caused by the mutual influence of the near fields of the dipoles [12]. The dense packing of dipole quantum dots, which are considered as RCs, in the matrix of an optical material makes it impossible to directly use mathematical approaches developed for calculating the linear effects of emission or multiple scattering of classical wave fields.

Meanwhile, the problem of light amplification with transformation of the temporal structure upon its reflection by an inverse medium can be solved by applying the concept of a near-surface layer with resonant polarization. In the case of such a low-dimensional layer, the polarization components of the light field are significant in the radiation reflected and acting within the layer on the inverted ensemble of atomic dipoles. The nonlinear components are caused by the so-called superradiant components of the medium's response to resonant radiation, calculated within the framework of quantum-mechanical equations of a two-level density matrix [13]. These components take into account the gain saturation and the phase nonlinearity of the array response, stimulated by a high concentration of active dipole centers, typical of dense media [14–16]. The action of both factors of the response nonlinearity can cause temporary instability of the initially continuous pump radiation amplified upon resonant reflection by the array and can be the cause of its modulation.

In this paper, we analyze the dynamic consequences of the near-field effect on the polarizability of active centers, resulting from the shift in the spectral gain line for resonant reflection of an inverted low-dimensional array of RCs. Solving the problem of transforming the temporal structure and achieving self-oscillation of the reflected light field under these conditions is important for the development of compact optical devices that generate coherent radiation.

Basic Equations

For the analysis of the dynamics of the plane-wave field in low-dimensional optical structures, the approximation of an ultrathin layer of resonant atoms with its inherent assumption of a longitudinally uniform field is convenient and acceptable [17]. The relationship between the fields is presented in the form of algebraic relations following from the electrodynamic conditions for Maxwell's equations. We consider an externally incident pump field wave with a stationary strength $E_i(t) = E_0$, a transmitted wave of a complex longitudinally uniform field acting in the medium with strength $E(t)$, and a reflected wave with strength $E_r(t)$. The dynamics of the acting and reflected fields are determined by non-stationary resonant variations in the nonlinear response of the RCs array medium interacting with the radiation. The material response, similar to the approach proposed in [18], is described in the present work by a variable gain n (proportional to the inverse population) and a complex amplitude of the polarization probability ρ. The time evolution of the resonant components of the response is analyzed within the framework of the formalism of optical quantum Bloch equations for an ensemble of two-level dipole RCs with an average electric moment μ. Therefore, the calculation scheme forms a system of nonlinear differential equations, and in the case of a stationary (quasi-continuous) external field $E_i(t) = E_0$, the system is autonomous.

The proposed and further considered modification of the kinetic equations is justified, for example, in [15]. In the scheme given below, the quasi-stationary intensities of the light fields E and E_r, as well as E_0, in normalized time $\tau = t/T_2$ are scaled as dimensionless variables (for example, $\mathrm{e} = \mu T_2 E/\hbar$, $\mathrm{e}_0 = \mu T_2 E t_0/\hbar$):

$$\frac{dR}{d\tau} = n\mathrm{e}_0 + (n-1)R - (\Delta\omega - \gamma n)S, \quad \frac{dS}{d\tau} = (\Delta\omega - \gamma n)R + (n-1)S,$$
$$\frac{dn}{d\tau} = \frac{\kappa - n}{\tau_{12}} - \kappa^2 \mathrm{e}_0 R - \kappa^2(R^2 + S^2), \quad \mathrm{e}_r(\tau) = -r\mathrm{e}_0 + R + iS, \quad -1 < n \le \kappa. \tag{1}$$

Here,

$R = \mathrm{Re}\rho$, $S = \mathrm{Im}\rho$ и n are the resonant polarization and gain variables,

$\Delta\omega = (\omega - \omega_0)T_2$ is the linear detuning of the carrier frequency of the light field ω from the center of the spectral gain line ω_0, normalized by the linewidth,

γ is the normalizing coefficient in the local Lorentz correction to the effective field (mainly determined by the ratio of the wavelength and film thickness),

κ is the gain index of the inversion layer, maximum at a given pump level,

r and t_0 t0 are the Fresnel reflection and transmission coefficients of the array,

$\tau_{12} = T_1/T_2$ is the ratio of the longitudinal (T_1) and transverse (T_2) relaxation times of the transition, or, for semiconductor quantum dots (QDs), the interband and intraband relaxation times, respectively.

System (1), in the uniform field approximation, characterizes the energy exchange between the input field and the quantum system forming the RCs array, taking into account the finiteness of the phase (transverse) relaxation time of the resonant polarization. The influence of excitation (pump current), which stimulates the inversion and determines its reversibility under the inevitable saturation during stimulated emission, is taken into account. Particularly characteristic is the consideration in scheme (1) of the dipole-dipole interaction inherent in dense resonant media, expressed by a local Lorentz correction to the field acting on the dipole active centers. The frequency detuning in the equation for polarization is expressed by the sum of the frequency defect $\Delta\omega$ and the Lorentz correction with the coefficient γ, the value of which can change as the inverse population saturates. For this reason, the nonlinear resonance detuning, which depends on the intensity of the radiation field, as a periodic violation of the resonant amplification condition, inevitably becomes a feedback factor in the generation circuit in the medium of the inverse array of RCs and can cause self-modulation dynamics of the process.

In the context of studying the dynamics of the nonlinear response of the array, which generates time variations in the field acting on the active centers, based on system (1), the problem of modeling the non-stationary interaction mode, which can lead to the occurrence of self-oscillations in the reflected radiation, is further solved.

Properties of Stationary Solutions

Nontrivial stationary solutions, determined from singular limits (1), depend nonlinearly on the amplitude e_0 and characterize the equilibrium states of the model. Formulating conditions for their dynamic stability or instability means assessing the possibility of oscillatory behavior of the reflected field in the physical situation where the external signal represents a rectangular field spike. It should also be assumed that the exposure time of such a light pulse with amplitude $e_i(\tau) = e_0$ should be comparable to or exceed the relaxation times T_1 and T_2.

Following the above, stationary solutions (1) ρ_S, n_S, corresponding to a certain level of continuous excitation e_0, can be expressed from algebraic expressions:

$$\rho_S = -\frac{n_S - 1 - i(\Delta\omega - \gamma n_S)}{(n_S - 1)^2 + (\Delta\omega - \gamma n_S)^2} n_S e_0, \quad \frac{\kappa - n_S}{\tau_{12} n_S} = \frac{\kappa^2 e_0^2}{(n_S - 1)^2 + (\Delta\omega - \gamma n_S)^2}. \quad (2)$$

Relations (2) represent a system of nonlinear algebraic equations for $R_S = \mathrm{Re}\,\rho_S$, $S_S = \mathrm{Im}\,\rho_S$ and n_S. The dependence of these quantities on the coefficients and parameters of system (1) is relatively easily calculated using parametric calculations. When constructing nonlinear dependencies on the level of normalized intensity e_0^2, the change in one of these variables, for example, n_S, is conveniently specified as a variable decreasing linearly within the limits (κ, 0).

To estimate the scale of the phenomena in the case of modeling based on the system of equations (1), approximate values of the parameters of real QDs -based emitting devices were used, which were taken from the scientific literature for QDs arrays in InAs/(Al)GaAs circuits on GaAs substrates [10, 19, 20]. Thus, the QDs density was in the range of (1.0…3.0) $\cdot 10^{11}$cm^{-2}. The dipole moment of an elementary emitter was estimated based on the approximate QDs size of 2…5 nm. The intensity of external radiation could reach a value of approximately $2.5 \cdot 10^5$ W/m^2 at wavelengths from the range $\lambda \sim (1.25 \ldots 1.3) \cdot 10^{-6}$ m; the linear detuning value $\Delta\omega$ was chosen within the line width; interband and intraband relaxation times is $T_1 \approx 1 \cdot 10^{-9}$ s, $T_2 \approx 1 \cdot 10^{-12}$ s.

Figure 1 shows the curves calculated parametrically based on relations (2), which characterize the change in the quantities ρ_S, n_S for different parameters of the system (1) depending on the excitation level e_0^2 – the normalized intensity of the external field. The particularly nonlinear nature of these dependencies is noted: within a certain range of variation of e_0^2, ambiguity is observed in the functions $n_S(e_0^2)$ and $|\rho_S(e_0^2)|^2$. In this case, we speak of bistability of states – the values of e_0^2 from this range correspond to three values of the functions, two of which correspond to the realized equilibrium states of the model, that is, at the same excitation level, the existence of two different sets of stationary values of the variables is possible.

Bistability is a special manifestation of the nonlinear relationship between variables and parameters. Nonlinearity is then caused by several interconnected physical mechanisms acting simultaneously. These mechanisms are capable of compensating or enhancing each other's role to varying degrees during the process, depending on the stage of the field-medium interaction and the combination of the nonlinear system's parameters. Bistability must manifest itself in the range of action of various factors altering the system's dynamics and under the condition of their different inertias. In this case, the change in gain occurs due to its saturation by the stimulated emission field and resonance detuning, which depends on the population difference, which decreases as saturation is approached.

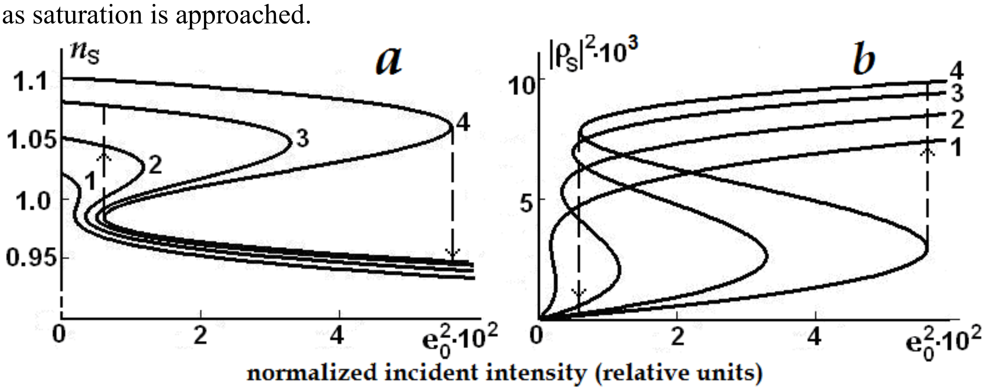

Figure 1 - Bistable dependences of stationary solutions of system (1) on the parameter of normalized intensity of external radiation;

$\kappa = 1.02$ (curve 1), 1.05 (2), 1.08 (3), 1.10 (4); $\gamma = 1.17$, $\Delta\omega = 1.0$, $\tau_{12} = 1.0 \cdot 10^3$

The transition from one set of stationary variable values to another occurs at the curve turning points. When the excitation level cyclically changes in the vicinity of these points, a hysteresis pattern appears in the dependence, with sharp jumps (usually referred to as "kinks") at these points. In the fragments of Figure 1, these transitions are indicated by vertical dotted lines only for curves 4.

This hysteresis-like nature of the relationship between state characteristics and excitation parameters suggests the possibility of abrupt state switches at the curve turning points. The dynamic model in this case is inherently unstable, with the potential for self-oscillations to develop at a constant excitation level. The calculation variants in Figure 1 demonstrate the criticality of the hysteresis loop width (the distance between turning points) relative to the maximum achievable gain level λ. Accordingly, at high gain levels, the consequences of instability are expected to be more widespread, even leading to the transition of the oscillatory energy exchange scheme to self-oscillations of the variables.

Assessment of the Dynamic Stability of the Equilibrium States of the Model

A qualitative study of the stability of solutions (1) near equilibrium states (2) makes it possible to evaluate the nature of its stability, determine the region of instability of solutions, and indicate the zone of parameters in which the process of transition to the equilibrium position takes the form of oscillations. The procedure for linearizing system (1) is reduced to replacing variables using their representation in the form: $R(\tau) = R_S + \Delta R(\tau)$, $S(\tau) = S_S + \Delta S(\tau)$, $n(\tau) = n_S + \Delta n(\tau)$ under the assumption that the components $\Delta R(\tau)$, $\Delta S(\tau)$ and $\Delta n(\tau)$.are relatively small. When formulating the linearized analogue of (1), higher powers of small components or their products are neglected, writing the following system:

$$\begin{aligned}
\frac{d}{d\tau}\Delta R &= (n_S - 1)\,\Delta R - (\Delta\omega - \gamma n_S)\,\Delta S - \left[n_S - 1 - \Delta\omega\,(\Delta\omega - \gamma n_S)\right]\frac{e_0}{X}\,\Delta n,\\
\frac{d}{d\tau}\Delta S &= (\Delta\omega - \gamma n_S)\,\Delta R + (n_S - 1)\,\Delta S + (\Delta\omega - \gamma)\,\frac{n_S e_0}{X}\,\Delta n,\\
\frac{d}{d\tau}\Delta n &= \kappa^2\left[n_S^2 - 1 - (\Delta\omega - \gamma n_S)^2\right]\frac{e_0}{X}\,\Delta R - 2\kappa^2\,(\Delta\omega - \gamma n_S)\,\frac{n_S e_0}{X}\,\Delta S - \frac{\Delta n}{\tau_{12}}.
\end{aligned} \tag{3}$$

here $X = (n_S - 1)^2 + (\Delta\omega - \gamma n_S)^2$.

Of further interest are the solutions for the components $\Delta R(\tau)$, $\Delta S(\tau)$ and $\Delta n(\tau)$, containing $\exp(\chi\tau)$ as the main part. The coefficient χ in the exponent of similar solutions of system (3) from a small neighborhood of stationary solutions (2) can be expressed as a real or complex value. In the latter case, the solutions will have not only an exponential but also an oscillatory nature.

The characteristic equation for the coefficient χ formed on the basis of the adopted algorithm is an algebraic cubic equation:

$$\begin{aligned}
&\chi^3 - \left[2\,(n_S - 1) - \frac{1}{\tau_{12}}\right]\chi^2 + \left[Y\,\frac{e_0^2}{X} + X - \frac{2}{\tau_{12}}\,(n_S - 1)\right]\chi + \frac{X}{\tau_{12}} - \\
&\left[(n_S - 1)\,Y - (\Delta\omega - \gamma n_S)\,(\Delta + \gamma)\,n_S\right]\frac{e_0^2}{X} = 0,
\end{aligned} \tag{4}$$

here $Y = 1 + \Delta\omega^2 + (1 - \gamma\Delta\omega)n_S$.

Solutions (3) that correspond to a specific range of values of its coefficients are considered special from the point of view of the correlation between the dynamic behavior of model (1) and the possible real time sweep of the radiation amplified by reflection. In this region, the characteristic equation (4), formulated on the basis of the linearized analogue of (3), can have one real and two complex roots (χ_1 and $\chi_{2,3}$):

$$\chi_1 = \sigma_- - \sigma_+ - \frac{1}{3\tau_{12}} - \frac{2}{3}(n_S - 1), \quad \sigma_\pm = \sqrt[3]{G \pm \sqrt{D}}\,,$$
$$\chi_{2,3} = -\frac{1}{3\tau_{12}} - \frac{2}{3}(n_S - 1) + \frac{1}{2}(\sigma_+ + \sigma_-) \pm i\frac{\sqrt{3}}{2}(\sigma_+ - \sigma_-), \tag{5}$$

here

$$D = G^2 + P^3,\ P = \frac{1}{9}\left[3\left(M - N + \Delta_S^2\right) - B\right],\ G = \frac{B^3}{27} - \frac{B}{6}\left(M - N + \Delta_S^2\right) + \frac{\Delta_S}{2}\left[\left(B - \frac{n_S e_0^2}{X}\right)\Delta_S + \gamma N\right],$$

$$B = n_S - 1 + \frac{1}{\tau_{12}},\quad M = \kappa^2\left(1 - n_S^2 + \Delta\omega\,\Delta_S\right)\frac{e_0^2}{X},\quad N = \kappa^2(1 + n_S)\frac{n_S e_0^2}{X},\quad \Delta_S = \Delta\omega - \gamma n_S.$$

Undamped periodic changes in the response variables $R(\tau)$, $S(\tau)$ and, accordingly, the normalized intensity of the reflected light field I, are possible for combinations of coefficient (1) values such that the real part of the roots $\chi_{2,3}$ (3) is positive. The points corresponding to the equilibrium states (2) in the phase space of system (1) then take the form of an unstable focus. Solutions (1), "starting" from the vicinity of such points and depicted by curves in the phase space of system (1), are represented by "unfolding" cyclic trajectories that leave the vicinity of points (2). At the same time, due to the inevitable saturation of the inverse population (amplification n) by stimulated emission, the generation power must stabilize. The curves are localized in a closed space, and their projections onto coordinate planes in phase space form limit cycles over time. In the temporal aspect, this dynamics of variables (1) will correspond to their self-oscillations arising spontaneously (at a constant level of stimulating factors - pumping and the amplitude of the excitation field strength e_i) - only for certain combinations of the values of the material parameters of the inverted layer and the excitation characteristics - the intensity e_0^2 and the frequency detuning $\Delta\omega$. The conditions for the existence of complex roots of equation (4) with positive values of their real part, following expressions (5), are formulated by the following relationships:

$$\sigma_+ + \sigma_- > \frac{2}{3\tau_{12}} + \frac{4}{3}(n_S - 1), \quad D > 0\ ; \tag{6}$$

The second requirement in (6) is a positive value of the discriminant D. This is the condition for the existence of complex roots of equation (4) in the form of $\chi_{2,3}$ (5).

Variants of parametric calculations based on relations (2), (5) of the typical dependence of the real and imaginary parts of $\chi_{2,3}$, corresponding to the attenuation (buildup) and frequency of oscillatory solutions (3) on the level of normalized intensity e_0^2 are presented in Figure 2. The instability region of solutions (3) on the scale of this parameter can be identified by the criterion Re $\chi_{2,3} > 0$. The buildup of small oscillations then occurs as an exponential increase in their amplitude over time. The corresponding exit of the phase curves from the vicinity of the singular points should occur at excitation intensity values located on the e_0^2 scale up to the intersection with the horizontal axis of the decreasing Re $\chi(e_0^2)$ dependences (curve 1 in Fig. 2, *a*) or up to the "right turning points" of the bistable curves (curves 2 - 4 in Fig. 2, *a*). The positions of these points on the e_0^2 scale are indicated in the figures by vertical dotted lines; the exit from the instability zone and the "transition" to the stability of the equilibrium state with an increase in e_0^2 should be abrupt.

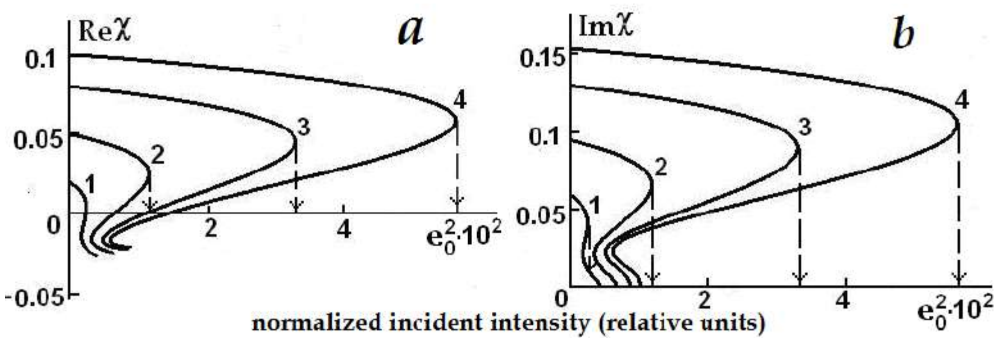

Figure 2 - Dependence of the real (a) and imaginary (b) parts of the complex roots of the characteristic equation on the parameter of the normalized intensity of external radiation;

$\kappa = 1.02$ (curve 1), 1.05 (2), 1.08 (3), 1.10 (4); $\gamma = 1.17$; $\Delta\omega = 1.0$, $\tau_{12} = 1.0 \cdot 10^3$.

An analysis of the curves indicates that the instability region on the e_0^2 scale expands with increasing maximum gain parameter κ; the dependences are calculated for its increasing values. The attenuation should decrease with increasing excitation, as should the oscillation frequency of the variables expressed by Im χ (Fig. 2b). However, at excitation values e_0^2 above the boundary, oscillatory solutions do not exist in the instability region. Therefore, the possibility of the emergence and development of regular oscillatory modes of the self-oscillatory type in the reflected radiation disappears.

Resonant Reflection Simulation Results

The qualitative analysis data indicate the range of parameters and initial conditions for the variables in which solutions (1) can be sought, describing self-sustaining pulsations in the intensity of the reflected light field. Therefore, within the framework of the numerical solution (1), it was further advisable to analyze the dynamics of the thin layer's response to a stationary external optical field.

Numerical integration of system (1) was carried out by the Runge - Kutta method for initial conditions that obviously correspond to the inverted state of the layer medium: for the population difference – $n(\tau = 0) = n_0$ (the value of n_0 for a given level of the exciting field amplitude e_0 was selected according to calculations illustrated by the $n_S(e_0^2)$ dependencies in Figure 1, a). For the polarization probability – $\rho(\tau = 0) = 0$ (it was assumed that the polarizing influence of external radiation is initially absent). The time dependence of the dimensionless intensity of the reflected radiation field $I_r(\tau) = |e_r(\tau)|^2$ was calculated.

Figure 3 shows examples of time sweeps of typical solutions (1) for a variable normalized intensity u(t), expressed in relative units on a nanosecond time scale. In the general case of a nonzero frequency defect (($\omega \neq \omega_0$), which causes linear phase modulation, the dipole-dipole interaction factor, which causes a shift in the resonant frequency and nonlinear phase modulation ($\gamma \neq 0$), is necessarily taken into account. The values of e_0 and the corresponding values of n_0 are mainly taken from the range of values in the instability zone of equilibrium states; in the variants of solutions (1) illustrated in Figure 3, line 2 in Figure 2 is chosen as the "reference" calculation curve defining this instability zone.

The time sweeps of the radiation in the fragments of Figure 3, a, b, d, are calculated for different excitation level values within the bistability region of equilibrium states. The $I_r(t)$ scans in fragments 3, d - f correspond to the same value for e0 with different gain levels κ. In general, these solutions describe an oscillatory transition to an unstable (quasi-stationary) equilibrium state, occurring with different relaxation rates.

The solution variants in Figures 3, c, c' are typical for calculations of $u(t)$ for values of e_0 outside the bistability zone (Fig. 3, c) and for e_0 corresponding to the stability region (Re $\chi_{2,3} < 0$). The formation of a regular regime with oscillations in the picosecond range thus occurs only in the bistability zone of equilibrium states; at a relatively low excitation level, the transition to the equilibrium state is purely relaxational in nature (Fig. 3, c). Outside the instability zone for the parameter e_0, the unfolding scenario presents a series of short relaxation bursts of intensity as a transition regime and, then, a relatively rapid transition to a stationary radiation level (Fig. 3, c’).

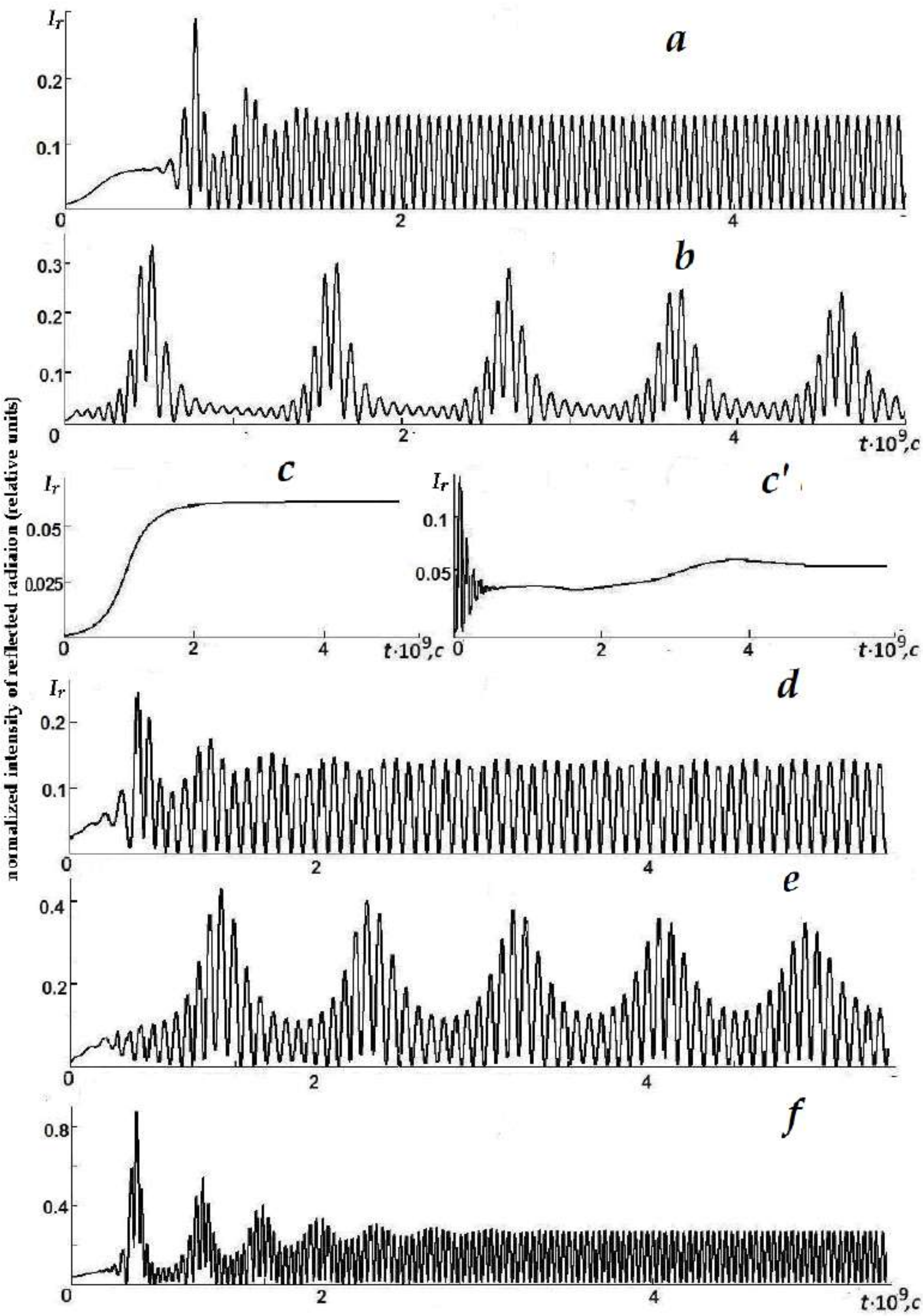

Figure 3 - Dependences of the normalized intensity of reflected radiation on time

$e_0^2 = 5 \cdot 10^{-3}$ (*a*), $7.5 \cdot 10^{-3}$ (*b*), $2 \cdot 10^{-3}$ (*c*), $1.0 \cdot 10^{-2}$ (*c*'), $6 \cdot 10^{-2}$ (*d* - *f*); κ = 1.05 (*a* – *d*), 1.07 (*e*), 1.10 (*f*); γ = 1.58, $\Delta\omega$ = 1.0, $\tau_{12} = 1.0 \cdot 10^3$.

In the "unstable" sweeps (Fig. 3, a, b, d - f), transient, relatively low-frequency oscillations with a nanosecond repetition period, commonly referred to as relaxation oscillations, are primarily distinguished. Their existence in kinetic processes is explained by the difference in transition probabilities in the pump and lasing channels. In this case, due to the superposition of relatively high-frequency oscillations, the transient pulsations $u(t)$ have a complex shape (Fig. 3, b, e), but their attenuation over time leads to the laser oscillatory system "entering" a quasi-stationary emission regime. At this stage of the process, higher-frequency intensity oscillations persist and assume a regular character. Their occurrence is explained by oscillations in the phase detuning of the polarization and field, caused by a resonance shift under the influence of the near fields of the dipoles. The high-frequency 'carrier' component is obviously nutational in nature – the polarization vector undergoes cyclical nutational movements around the field direction. The gain, due to the resonance shift, which can assume a periodic nature, is also drawn into an oscillatory mode. Nonlinear high-frequency pulsations of intensity $u(t)$ with a stationary envelope, representing modulation of the reflected signal after the transient stage, occur due to these relatively weak self-oscillations of the gain $n(t)$.

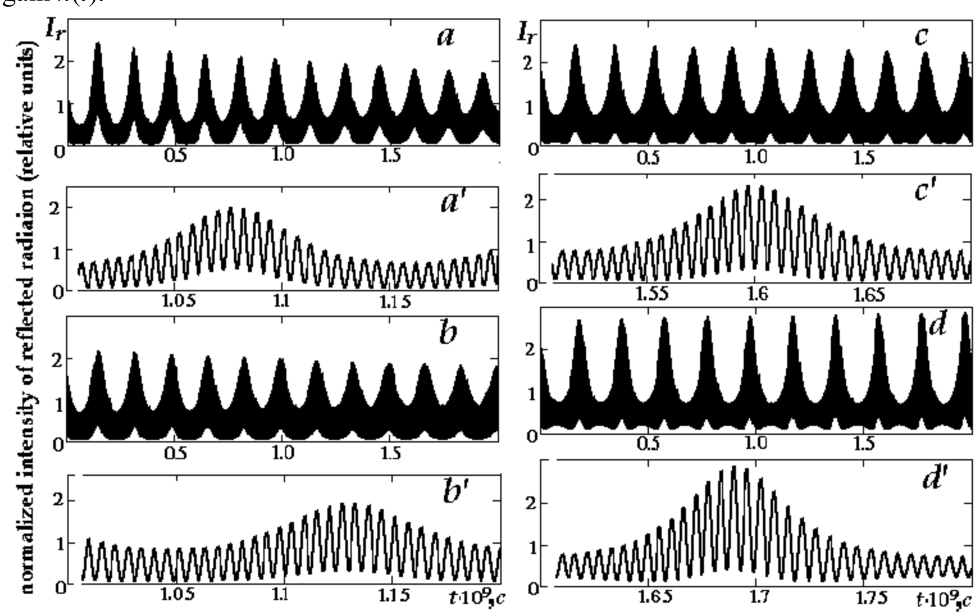

Figure 4 - Dependence of normalized radiation intensity on time

$\Delta\omega$ = 1.0 (*a*), 0.5 (*b*), 0.2 (*c*), 0.02 (*d*); $e_0^2 = 5{\cdot}10^{-2}$, κ = 1.08; γ = 1.57, $\tau_{12} = 1.0{\cdot}10^3$.

At the stage of stabilization of the intensity envelope dependence, its "carrier" high-frequency oscillations are regularized, creating a picture of the self-oscillatory process in the radiation. In the subsequent time of development of the resonant reflection process before the cessation of the action of the exciting field and pumping, the oscillation amplitude tends to a constant level approximately as illustrated by the dependencies in Figures 3, a, f). The duration of the transient regime with losing contrast and damping low-frequency oscillations $u(t)$ mainly depends on the value of the ratio of relaxation times τ_{12} and the unsaturated gain parameter κ. The frequency of the "carrier" oscillations depends on the amplitude of the exciting field e_0 from the bistability zone and on the gain parameter κ. Qualitatively, the latter dependencies coincide with those predicted by calculating relations (2), (5), characterizing the solutions of system (3). Figure 4 shows an example of modeling the process of stabilizing the low-frequency components of the temporal structure as the linear detuning changes (fragments a - d of the figure). The formation of a series of periodic intensity spikes of a relaxation nature with a 'carrier' nutation component, illustrated in fragments of Fig. 4, a' - d' with higher resolution, occurs as the external excitation frequency approaches resonance.

Conclusion

An oscillatory system formed by a low-dimensional array with a high RCs density and external resonant radiation, due to the loss of coherence of field oscillations and polarization, is thus capable of transitioning to an unstable equilibrium state in which self-oscillations of variables occur, i.e., to a special quasi-stationary (nonlinearly modulated) state. Physically, this corresponds to achieving a self-oscillation regime (regular pulsations) during resonant reflection and signifies the possibility of converting a light signal with a constant intensity envelope into a time-modulated signal, with the modulation parameters controlled by changes in light power.

The result of this analysis is the study of the current problem of light wave behavior at an interface between media containing a thin layer of material with a relatively strong nonlinear response to radiation in a frequency range close to one of the optical resonances. Modeling and analytical evaluation of the kinetics of stimulated emission in the approximation of a thin layer of an inverse medium allowed us to characterize the dynamic phenomenon of amplitude self-modulation of light reflected by a low-dimensional array of RCs. This effect can be exploited in the development of extremely compact quantum dot laser devices. Such devices should offer certain advantages over existing ones. Indeed, in the phenomenon of light amplification by reflection, the interaction of light with the gain medium can occur primarily in a thin layer near the interface between the media, with little radiation penetration into the gain medium. This reduces the technical requirements for the optical properties of the gain media, such as their homogeneity, transparency, and so on. Some potential devices utilizing the phenomenon of light amplification by reflection have already been implemented, but for inverted layers with a relatively high concentration of RCs, the nonlinear processes that stimulate spontaneous modulation have not yet been studied in detail.

In the IR frequency range, electro-optical materials capable of applying standard methods of Q-switching pulsed lasers and shortening pulse durations are still lacking. Therefore, laser generation research is currently being intensively developed in relation to technologies for generating regular trains of short and ultrashort pulses with controlled temporal parameters in this spectral range. The results of the reflection dynamics calculations presented in this article, taking into account the self-phase modulation of the light field, will be useful for developing methods for generating trains of short light pulses with relatively low average intensity in compact laser devices based on materials with quantum-well effects.

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Conflict of interest. The authors declare that they have no conflicts of interest.

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Тимощенко Е. В., Юревич В. А., Хомченко А. В., Юревич Ю. В. **РЕЛАКСАЦИОННАЯ СТРУКТУРА ОТРАЖЕНИЯ СВЕТА НИЗКОРАЗМЕРНЫМ МАССИВОМ РЕЗОНАНСНЫХ КВАНТОВЫХ ИЗЛУЧАТЕЛЕЙ**

Предложена модификация системы уравнений Максвелла Блоха для изучения осцилляций интенсивности когерентного излучения при отражении от низкоразмерного массива, образованного структурой из резонансных дипольных центров относительно высокой плотности. Моделирование и качественный анализ предложенной системы кинетических уравнений показали, что вариации фазового соотношения светового поля и резонансной поляризованности массива вызывают внутреннюю неустойчивость физической системы энергообмена, что проявляется в выраженной осцилляторной структуре отраженного излучения. В моделировании использованы материальные характеристики полупроводниковых квантовых точек.

Ключевые слова: квантоворазмерные структуры, диполь-дипольное взаимодействие, резонансное отражение, оптическая бистабильность, самопульсации излучения.